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# USING THE DSST SEMI-ANALYTICAL ORBIT PROPAGATOR PACKAGE VIA THE NONDYWEBTOOLS/ASTRODYWEBTOOLS OPEN SCIENCE ENVIRONMENT

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An initiative within the realm of Space Surveillance Awareness (SSA) is to create an open source software suite that can provide all space actors access to the basic SSA analysis tools needed to operate safely and efficiently in space. These applications include observation compression, orbit propagators, state transition matrix, weighted least squares, improved nonlinear Kalman filters, realistic process noise, atmosphere density variations, observation data association and observation data simulation which will be rewritten in a modern distributed object-oriented computing environment. It is the intent to make these applications useable via an Internet framework. While the project is being developed, one of the tools considered, the DSST standalone, which is an accurate Semi-Analytical Satellite Theory, can be used through Internet framework $\mathrm{Astrody}^{\mathrm{Web}}_{\mathrm{Tools}}$. This provides an Astrodynamics framework so as to carry out open science in which specialized applications can be integrated, and to encourage scientific collaboration through Internet. This framework, through a user-friendly web interface, allows the user to choose applications, introduce data and select appropriate constraints in an intuitive and easy way, with the help of graphical interface options. After that, the application is executed in real-time and the critical information about program behavior and output (graphical representation of data, statistical analysis or whichever manipulation therein) is shown via the same web interface and can be downloaded to users' computers.

## I. INTRODUCTION

Hundreds of thousands of human-made objects are orbiting at different altitudes around Earth. These objects include active and inactive satellites, spent rocket bodies used to launch satellites, and a wide variety of objects of all shapes and sizes, which have been released during missions or have come from collisions, explosions, and so on. Tracking them as precisely as possible, and analyzing their potential impact on space activities, is an essential part of Space Surveillance Awareness (SSA).

The accuracy of the SSA orbital data products are affected by the evolution of the sensors feeding the network, by the knowledge and control of the errors in the sensor network, by the knowledge of the space environment, by the available computing resources (both hardware and software), and by the number of space objects to be monitored. However, the quality and quantity of the orbital data products available to space operators has evolved slowly over time. Further, error analysis of key issues is still in flux. The Iridium /Cosmos collision event in 2009 demonstrated that accurate SSA data was not sufficiently integrated into owner/operator management of satellite missions[1]. Further, lack of data fusion and inadequate modeling of the satellite motion are technical factors contributing to erroneous orbit prediction of the close approach distances in many instances.

In 2010, P. J. Cefola, B. Weeden and C. Levit[2] presented a plan for an "Open Source Software Suite for Space Situational Awareness and Space Object Catalog Work." This paper addressed two major issues:

1. the problem of adapting legacy SSA software tools to modern computing environments
2. the addition of the new analytic functionality to the operational SSA toolbox

Adapting complex scientific software to modern computing environments includes:

- non-invasive encapsulation of legacy binaries
- migration of SSA tools to a language platform employing object-oriented and component technologies such as C++
- a plan for creating a Web 2.0 architecture

Observation compression, certain orbit propagators, improved nonlinear Kalman Filters all are examples of 'new functionality' for the operational SSA environment.

For example, this project currently intends to include a variety of orbit propagators: Numerical Integration (Special Perturbations), DSST[3], Brouwer-Lyddane[4], NORAD GP[5] (SGP, SGP4, SGP8, with tesseral m-daily option), NORAD HANDE[6] (with tesseral m-daily option) NAVSPASUR PPT[7,8] (with tesseral m-daily option), and the Russian A, AP, and NA theories[9].

While the re-engineering processes of these orbit propagators is being carried out, at the same time we are working on their integration into a Web framework, which will allow the scientific community free use of these tools through the Internet.

One of these orbit propagators is DSST (Draper Semi-Analytical Satellite Theory) which was developed by P. J. Cefola, W. McClain, L. Early, R. Proulx, M. Slutsky and their colleagues at the Computer Sciences Corporation and the Charles Draper Laboratory (CSDL) in the 1970s and 1980s. In its development at the CSDL, DSST also benefited from numerous enhancements made by Massachusetts Institute of Technology graduate students under the direction of the CSDL staff. DSST was developed with an emphasis on accuracy and computational efficiency. The semi-analytical theory has been used extensively to study the long-term evolution of orbits. The constants in the semi-analytical theory, the mean orbital elements, have also proven quite useful as the solve for variables in both batch least squares and recursive filter orbit determination processes based on the semi-analytical theory. The historical evolution of DSST can be seen in Reference[2].

Nonlinear Dynamics Web Tools, $\text{Nondy}^{\text{Web}}_{\text{Tools}}$ [10], is an e-Science project being developed in the University of La Rioja. The aim of this project is to build a non-commercial infrastructure to encourage scientific collaboration in which specialized software tools may be used freely through Internet. This project should promote not only e-Collaboration among the Web-Site's users, but also optimize use of resources, both human and material. This project is not a closed one, but open to the collaboration of whoever wishes to participate in this initiative. $\text{Astrody}^{\text{Web}}_{\text{Tools}}$ [11] is the specialized part related to Astrodynamics and Celestial Mechanics.

A brief description of the DSST standalone characteristic is given in this paper. Then the $\text{Astrody}^{\text{Web}}_{\text{Tools}}$ framework is presented. Finally we describe the integration of DSST into the Web-site and show its use through the Internet.

## II. DSST

DSST[12,13,14,15,16,17,18] is based on a semi-analytical satellite theory expressed in nonsingular equinoctial elements[19,20], which allows taking advantage of the accuracy of Special Perturbations (numerical integration) and the efficiency of General Perturbations (analytical satellite theory) because the influence of the long and short-period perturbations can be decoupled using the Perturbation Theory, and thus handled separately. Note that the equinoctial elements are related to the classical Keplerian elements by the following equations:

$$\begin{aligned} a &= a \\ h &= e \sin(\omega + \Omega) \\ k &= e \cos(\omega + \Omega) \\ p &= \tan(i/2) \sin(\Omega) \\ q &= \tan(i/2) \cos(\Omega) \\ \lambda &= M + \omega + \Omega \end{aligned} \qquad [1]$$

The equinoctial elements given in Eq. (1) represent one possible choice for the nonsingular state variables[21].

The Generalized Method of Averaging leads to the Lagrangian VOP form given in Eq.(2) which is useful for perturbations with a conservative potential and the Gaussian VOP form (Eq.(4) in Ref[3]).

$$\frac{d\bar{a}_i}{dt} = \sum_{j=1}^{6} (\bar{a}_i, \bar{a}_j) \frac{\partial}{\partial \bar{a}_j} \left[ \frac{1}{2\pi} \int_0^{2\pi} R dM \right] \qquad [2]$$

The Gaussian VOP is very useful for atmospheric drag and solar radiation pressure perturbations where it may be difficult to express the RHS's of the equations of motion in orbital elements.

Large stepsizes are possible for both the Lagrangian and Gaussian VOP forms. The Gaussian VOP may be uniquely able to take advantage of parallel processing concepts.

The atmosphere drag and solar radiation pressure models referenced in Tables 1 and 2 employ the Gaussian VOP.

The statement "DSST computes the short periodic motion using Finite Fourier series" applies to both Lagrangian VOP and Gaussian VOP.

Force models currently considered in DSST for the mean element equations of motion and for short periodic motion are shown in Tables 1 and 2. Refinement of the $J_2$-squared model to better address high eccentricity cases is currently being investigated by Z. Folcik and P. Cefola.

| Perturbation factors | Mean element equations of motion |
|---|---|
| Zonal harmonics including $C_{20}$ | Linear terms in general form |
| Second-degree zonal harmonic | Second-order terms for the a, h, k, p, q, $\lambda$ rates<br>Terms of order $J_2^2\ e^2$ neglected |
| Tesseral lm harmonics of geopotential (2<l<50, 1<m<50) | Linear terms, including resonance effects in general form. Modified expansion for the Hansen coefficients |
| Attraction of the Moon and the Sun | Linear terms in general form |
| Atmosphere drag | Linear and cross with $C_{20}$ terms. Rates evaluated via quadratures (Harris-Priester, Jacchia-Roberts[22], MSISE-90) |
| Solar pressure | Linear terms of direct solar pressure. Rates evaluated via quadratures. Cylindrical model for shadow |
| Solid Earth Tides | Love number term |

Table 1: DSST mean element equation of motion

DSST also includes a semi-analytical theory for the partial derivatives of perturbed motion and an efficient interpolation strategy, which greatly assists in producing the perturbed position and velocity and the partial derivatives at output request times for a given satellite.

GTDS R&D is an orbit determination system with orbit propagators, observation sensor models, and nonlinear estimation technology. DSST is one of the several orbit propagators that are available within GTDS. Other orbit propagators available within GTDS include Cowell numerical integration (fixed time step), Cowell numerical integration (time regularized), Brouwer, Brouwer-Lyddane, NORAD GP Theories, and NAVSPASUR PPT2.

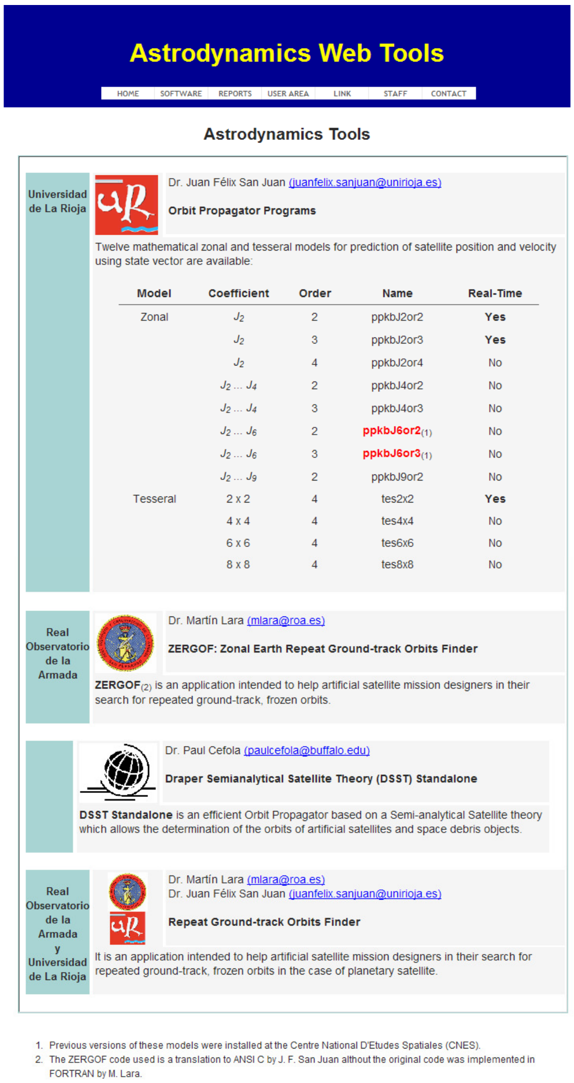
The Cowell orbit propagators are general with respect to the physical models. Brouwer, Brouwer-Lyddane, the NORAD GPs, and NAVSPASUR PPT2 all have similar physical models: secular and long-periodic motion due to $J_2$ through $J_4$ and short-periodic motion due to $J_2$. The DSST includes physical modeling that is nearly as general as Cowell.

| Perturbation factors | Short periodic motion |
|---|---|
| Zonal harmonics including $C_{20}$ | First-order terms in a, h, k, p, q, $\lambda$ treated via a closed-form expansion in true longitude |
| Second-degree zonal harmonic | Second-order terms in a, h, k, p, q, $\lambda$ treated via an expansion in true longitude<br>Terms of order $J_2^2\ e^2$ neglected |
| Tesseral lm harmonics of geopotential (2<l<50, 1<m<50) | Linear terms in general form – partitioned into three categories<br>1. 'm-dailies'<br>2. linear-combination terms<br>3. $J_2$/tesseral m-daily coupling terms |
| Attraction of the Moon and the Sun | Linear terms in general form. Closed form expansion in eccentric longitude. Weak time dependent terms |
| Atmosphere drag | Linear terms via an expansion in mean longitude. Coefficients evaluates via quadratures |
| Solar pressure | Linear terms via an expansion in mean longitude. Coefficients evaluated via quadratures |
| Solid Earth Tides | No |

Table 2: DSST short periodic motion

## III. ASTRODYNAMICS WEB TOOLS

The current prototype of $\text{Astrody}^{\text{Web}}_{\text{Tools}}$ allows that through a user-friendly web interface, the user chooses the application, introduces the data and selects the appropriate constraints in an intuitive and easy way, with the help of the options in the graphical interface. After that, the application is executed in real-time and the critical information about the program behavior and the output (graphical representation of the data, statistical analysis or any manipulation thereof whatsoever) are shown via the same web interface or downloaded to his computer.




| Model | Coefficient | Order | Name | Real-Time |
|---|---|---|---|---|
| Zonal | $J_2$ | 2 | ppkbJ2or2 | **Yes** |
| | $J_2$ | 3 | ppkbJ2or3 | **Yes** |
| | $J_2$ | 4 | ppkbJ2or4 | No |
| | $J_2 \ldots J_4$ | 2 | ppkbJ4or2 | No |
| | $J_2 \ldots J_4$ | 3 | ppkbJ4or3 | No |
| | $J_2 \ldots J_6$ | 2 | **ppkbJ6or2**(1) | No |
| | $J_2 \ldots J_6$ | 3 | **ppkbJ6or3**(1) | No |
| | $J_2 \ldots J_9$ | 2 | ppkbJ9or2 | No |
| Tesseral | 2 x 2 | 4 | tes2x2 | **Yes** |
| | 4 x 4 | 4 | tes4x4 | No |
| | 6 x 6 | 4 | tes6x6 | No |
| | 8 x 8 | 4 | tes8x8 | No |



Fig. 1: Available applications in $\text{Astrody}^{\text{Web}}_{\text{Tools}}$

Figure 1 shows the four current applications available in $\text{Astrody}^{\text{Web}}_{\text{Tools}}$. Besides DSST, the others application are:

- Orbit Propagator Programs[23,24] is an application, which contains twelve Analytical Orbit Propagator Programs (AOPP). These orbit propagators calculate the orbiter's position and velocity at any given moment directly by means of a function of time and the initial position and velocity of the orbiter. Every AOPP has been automatically obtained from an analytical approximation to the artificial satellite problem using a code generator. The AOPPs have been classified first by gravity coefficients (i.e. zonal or tesseral), then by the selected coefficients and finally by the accuracy of the analytical approximation used.
- ZERGOF[25] (Zonal Earth Repeat Ground-track Orbits Finder) is a software package designed to search for repeating ground-track orbits automatically, in the case of the Earth, in a proportion between the number of nodal periods of the satellite on its orbit and the number of nodal days previously selected by the user. These kinds of orbits are highly desirable as nominal orbits for a variety of missions for artificial satellites, because their eccentricity and argument of the perigee remain almost constant or *frozen* during a long period of time.
- Repeat Ground-track Orbits Finder is a software package designed to search for repeating ground-track orbits automatically in the case of a planetary satellite. The search for an orbit of this kind begins when an approximate solution is obtained by means of analytical techniques. Subsequently, the above approximate solution is refined by using numerical continuation methods. The first part of this process is made using a *Mathematica* package, whereas the second is available in our Web-Site at this moment.

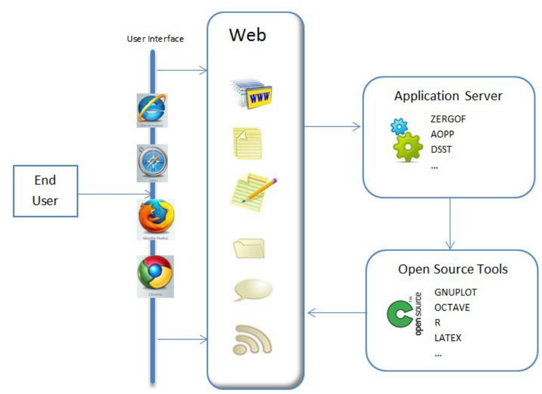



Fig. 2: Basic architecture of $\text{Astrody}^{\text{Web}}_{\text{Tools}}$

Figure 2 shows an overview of the architecture proposed for $\text{Astrody}^{\text{Web}}_{\text{Tools}}$. Through our framework the registered user can select and execute one of the available applications after completing the appropriate form with the initial values and parameters. The user-data are converted into the input of the application, which is executed, and the results are stored in files. After that, the user-results are processed by other open source applications such as Gnuplot, Octave, the statistical package R, Latex, or any other needed applications, and their outputs are embedded in the web page. In addition, the graphics, pdf reports, and other outputs, can be downloaded directly by the user in several formats —for instance, the images can be obtained in eps or pdf format.

For example, in the case of ZERGOF, the search for a frozen orbit begins when the user selects its characteristics. The gravity model, the number of zonal harmonics, the repeat ground track constrain and the type of periodic orbit, which is classified in near-circular Sun-Synchronicity orbit, near-circular with a selected inclination orbit, critical-inclination Sun-Synchronicity with argument of the perigee $90^0$ or $270^0$ orbit, or critical-inclination with an argument of the perigee $90^0$ or $270^0$ and a selected eccentricity orbit. The application obtains an approximate solution using analytical techniques. Subsequently, this solution is refined by means of numerical continuation methods.


Fig. 3: ZERGOF Web interface

Figure 3 shows an example of the use of ZERGOF. This searches the initial conditions of an almost circular orbit with a $56^0$ orbit inclination, where 5 revolutions in 3 days is the repeat ground track constraint. The model only considers between $J_2$ and $J_9$ zonal harmonics.

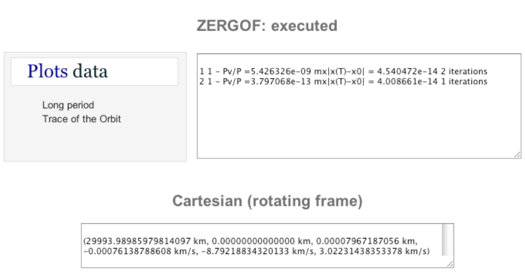



Fig. 4: ZERGOF output

Figure 4 shows the results provided by the Web-site after the execution of the application. The links in the left part of the interface facilitate graphical displays of the obtained results.

The trace of the orbit can be seen in Figure 5, whist Figure 6 shows the evolution of the long-period terms.

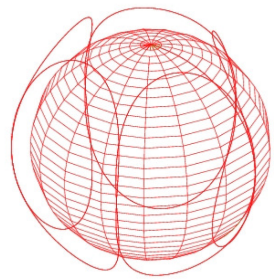


Fig. 5: Satellite trace

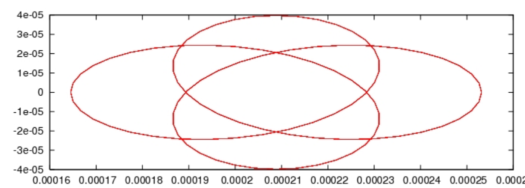



Fig. 6: Long-period terms $(e \sin g, e \cos g)$

## IV. USER DSST THROUGH ASTRODYNAMICS WEB TOOLS

DSST is a Fortran 77 based application, which does not have any type of graphical user interface (GUI), thus its execution is made from a command line. Currently, the DSST standalone source code is maintained by Z. Folcik and P Cefola. Figure 7 shows the basic DSST execution. The application reads the physical model files, which contain:

- astrodynamics constants data
- Solar/Lunar/Planetary ephemerides in either the J2000 or true data coordinates
- time conversion coefficients and polar motion coefficients
- 50x50 geopotential models
- quasi-logarithmic planetary geomagnetic indices and night-time minimum exospheric temperatures for the Jacchia-Roberts atmospheric density model

and the user-data files, which contain the initial conditions of the satellite and the configuration parameters of DSST and the output times selected by the user. Once data and constants are stored in memory, DSST carries out the calculus and the outputs are stored in two files.

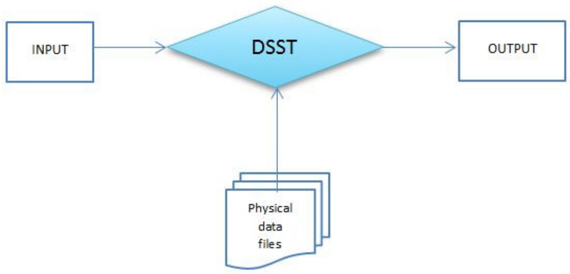



Fig. 7: DSST executed mode

This executed mode represents a classical pattern in scientific programming, which we could call a black-box paradigm. The application is treated like a black box, as users only need to modify some input data files so as to run the application without re-compiling each time. Our Web-site provides these applications with an easy-to-use graphical-user web-interface which simplifies communication with the applications, such that their integration into the $\text{Astrody}^{\text{Web}}_{\text{Tools}}$ framework is almost immediate, and then the details of their implementation, programming languages used, and so on, can be relegated to a secondary plane.

The current DSST Web interface allows the user to introduce the epoch, epoch mean elements set (Keplerian or Equinoctial), dynamical parameters, and so on, in two basic ways: the user-data file can be uploaded to $\text{Astrody}^{\text{Web}}_{\text{Tools}}$, as Figure 8 shows, or the user can fill in a web form, as Figure 9 shows.

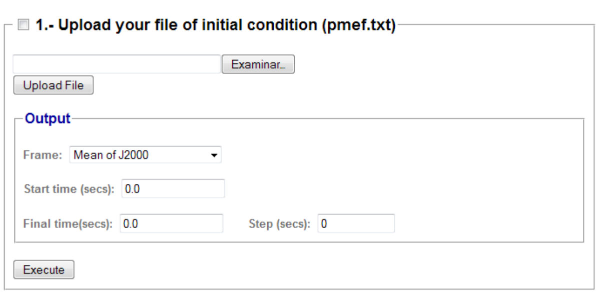



Fig. 8: Upload user-data file

Figure 10 shows the results provided by the Web-site after the execution of DSST. The information shown is obtained from the two output files generated by this application: dsst.output and SPGOUT. The first contains the output data. The second is a text file, which stores some of the DSST force model parameters used in its execution and some of its intermediate output: element rate and short period model partial derivative parameters, as well as short-periodic model options and short-periodic Fourier coefficients.

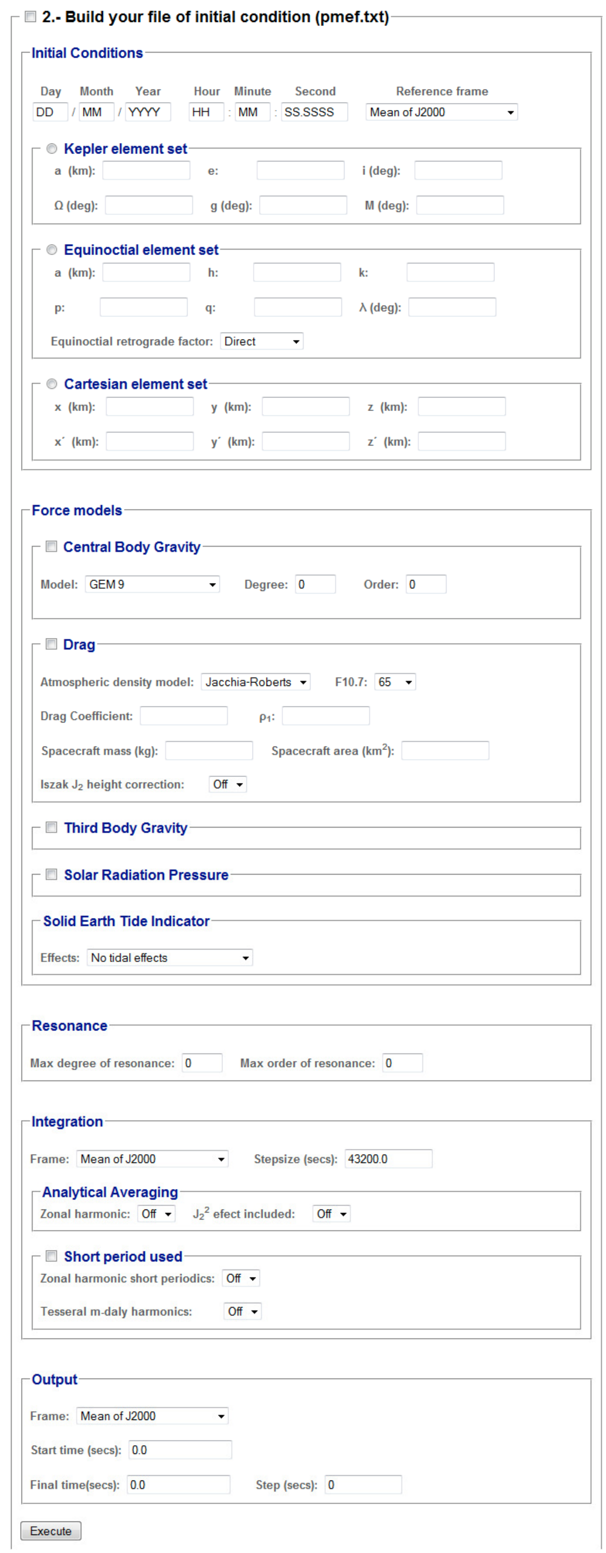



Fig. 9: DSST Web form

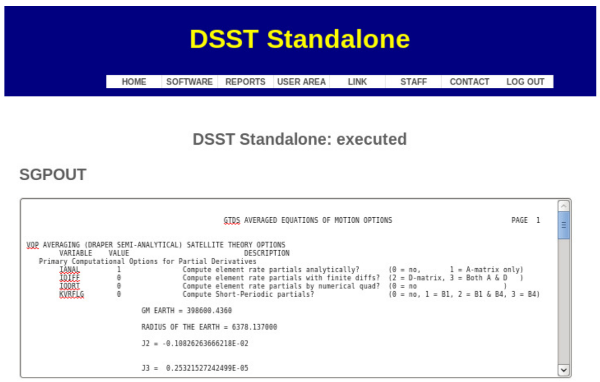



Fig. 10: DSST Web form

## V. CONCLUSION AND FUTURE WORKS

The creation of an open source software suite of basic SSA tools available to all space actors is an important step towards enhancing safe and efficient operations in space. While the project is being developed, one of the tools considered, DSST standalone, can be used through Internet. DSST is an accurate, efficient and extensively used Semi-Analytical Satellite Theory. Astrody$^{Web}_{Tools}$ is the Web framework, which provides DSST with a friendly GUI and users with a web service through which they can easily access using a Web browser.

In our project, DSST users play a relevant role in the improvement of this Web service, which is generated from the DSST output, because it is possible to modify the DSST Web interface in function of users' needs. For example, download files in a specific format, generate all kinds of graphics, allow uploading a file with their data in order to compare with the DSST output, or whatever users need. This collaboration intends to maximize the full utility of DSST output.

We are developing a new web environment, which will replace the current prototype. This will be available by the end of 2011 and Web 2.0 facilities and e-Learning tools will be added, in order to organize courses centered on these applications for those researchers and students who are interested in learning the theoretical knowledge they are based on.

## VI. ACKNOWLEDGMENTS

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