MINIMUM-TIME, CONSTANT-THRUST ORBIT TRANSFERS WITH NON-CIRCULAR BOUNDARY CONDITIONS

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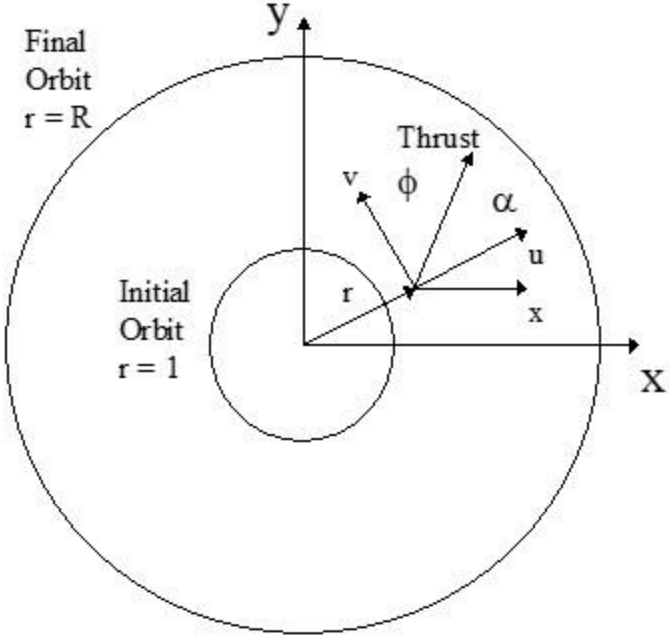
Background

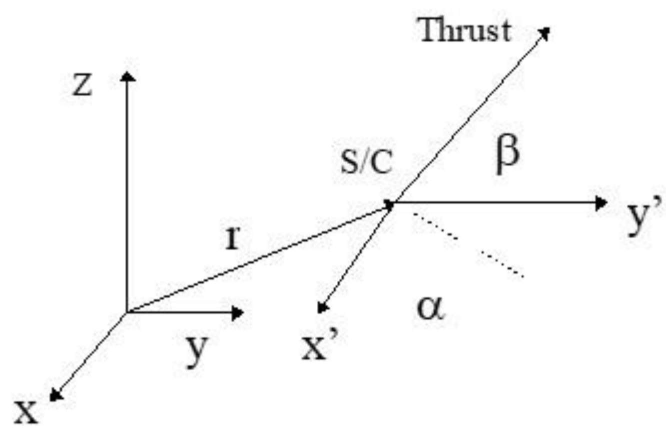
- Efficient orbital maneuver capability will be an important component of any in-space debris mitigation concept, whether it is to match velocities with a resident space object, or to match a required state prior to intercept.
- Optimization of the continuous orbital maneuvers using electric propulsion is not as straightforward as in the impulsive case, and guaranteed optimization requires classical indirect methods.
- Time-optimal control laws for orbital maneuvers can be produced by solving the associated calculus of variations problem using Euler-Lagrange theory in order to minimize the trip time between orbital states.
- Indirect methods of optimization can be used for complicated orbital transfers using the continuation method, as an alternative to direct methods such as collocation approximations

Problem Geometry Using Canonical Units

- The scalar distance from the origin is r
- The time rate of change of r is u, where u = dr/dt
- The local horizontal component of velocity is v, perpendicular to r
- **The polar angle does not appear in the equations of motion, freeing one constraint**
- For the 2D case, initialization requires t_f, λ_u and λ_v



- For the 3D case, initialization requires t_f and four Cartesian costates (related to 2D case)
- 3D end conditions can include angular momentum components



Optimal Control Formulation

- Equations Of Motion In Polar Coordinates:

$$\dot{r} = u$$

$$\dot{u} = \frac{v^2}{r} - \frac{\mu}{r^2} + A \sin \phi$$

$$\dot{v} = -\frac{uv}{r} + A \cos \phi$$

- The Hamiltonian and cost functional (time):

$$H = L + \bar{\lambda}^T \bullet \bar{f}, \quad J = \int_{t_0}^{t_f} L \, dt, \quad L = 1$$

Optimal Control
Formulation, Continued:

- Hamiltonian, Costates, and Control Law to be used in a numerical shooting method:

$$H = 1 + \lambda_r u + \lambda_u \left(\frac{v^2}{r} - \frac{\mu}{r^2} + A \sin\phi\right) + \lambda_v \left(-\frac{uv}{r} + A \cos\phi\right)$$

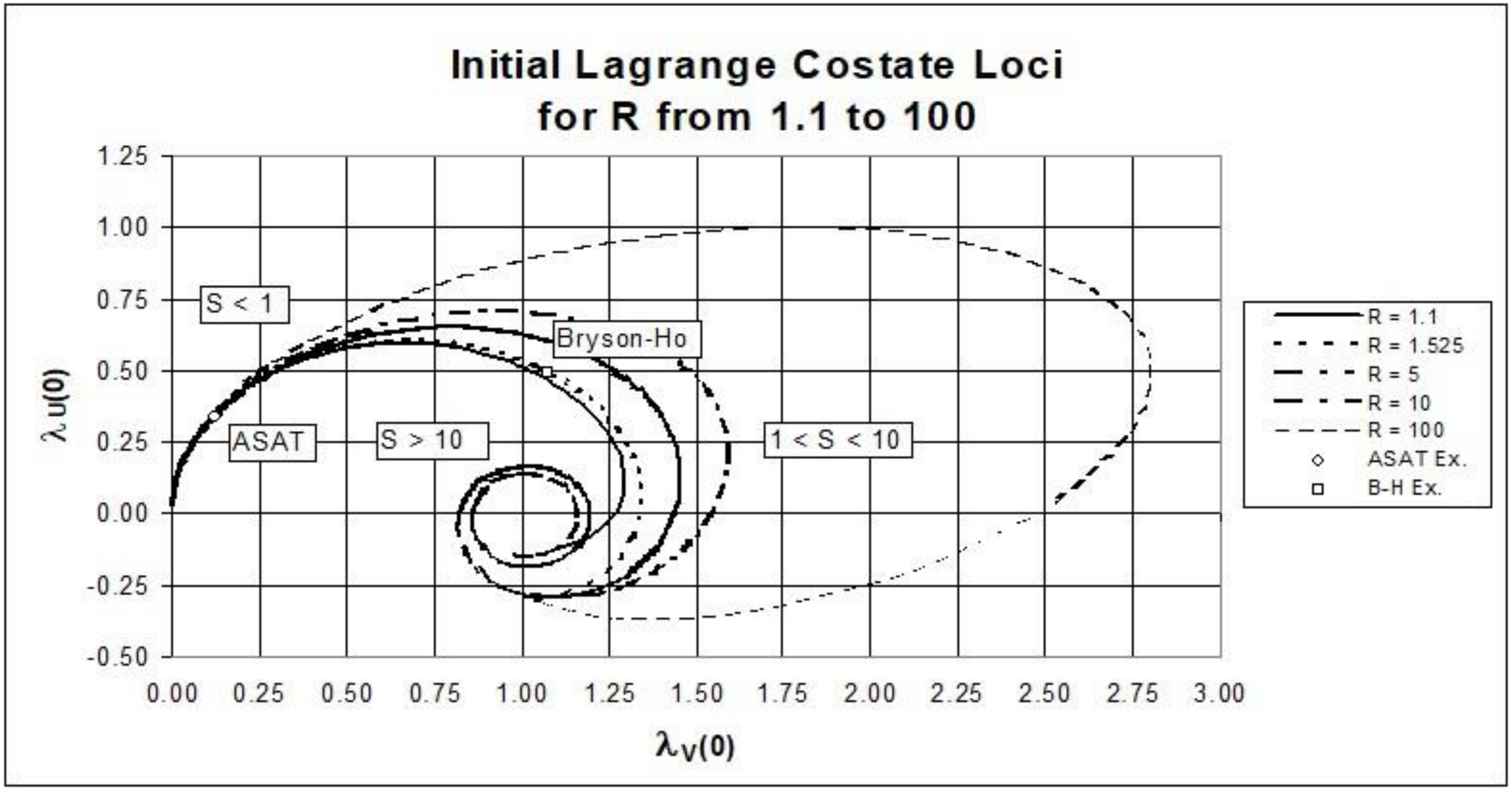
$$\dot{\lambda}_r = -\frac{\partial H}{\partial r} = -\lambda_u \left(-\frac{v^2}{r^2} + \frac{2\mu}{r^3}\right) - \lambda_v \left(\frac{uv}{r^2}\right)$$

$$\dot{\lambda}_u = -\frac{\partial H}{\partial u} = -\lambda_r + \lambda_v \left(\frac{v}{r}\right)$$

$$\dot{\lambda}_v = -\frac{\partial H}{\partial v} = -\lambda_u \left(\frac{2v}{r}\right) + \lambda_v \left(\frac{u}{r}\right), \quad \tan\phi = \left(\frac{\lambda_u}{\lambda_v}\right)$$

Initial Costate Model:
Regions Characterized by S Values

- Optimal Initial Costates, R = 1.1 To 100, 2D Case:



Initial Costate Model Formulation

- For high thrust to gravity ratio, or for short arc transfers:

$$t_f = 2S, \quad \lambda_u(0) = S, \quad \lambda_v(0) = S^2$$

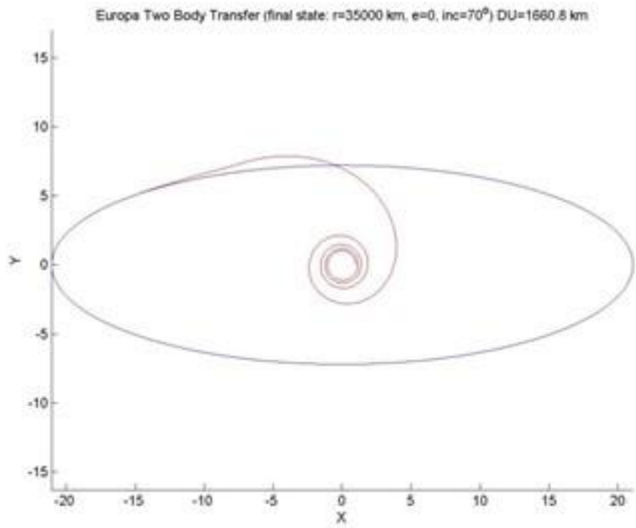
- Where the quantity S is defined as:

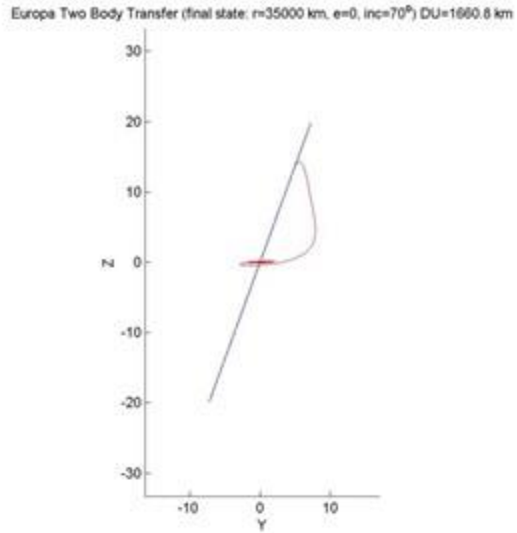
$$S \equiv \sqrt{\frac{R-1}{A}}$$

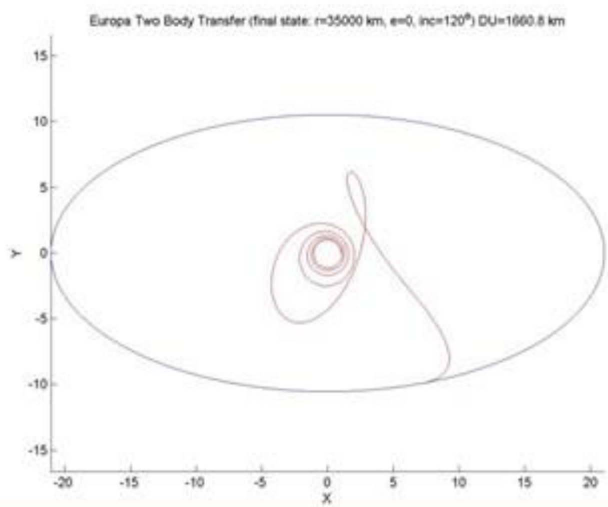
Numerical Examples

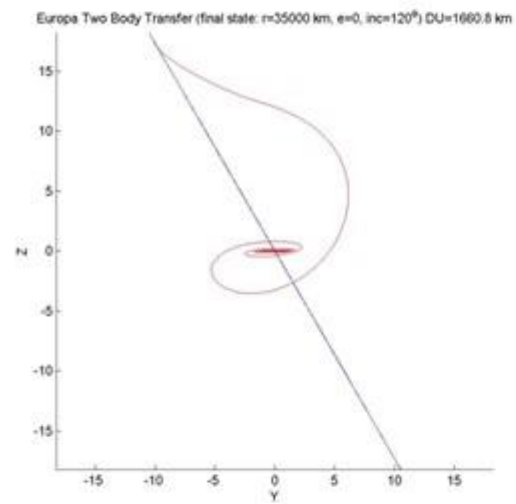
Europa Science Orbit Transfer

Final Inc = 70 & 120 deg, Acc = 0.01 (canonical)



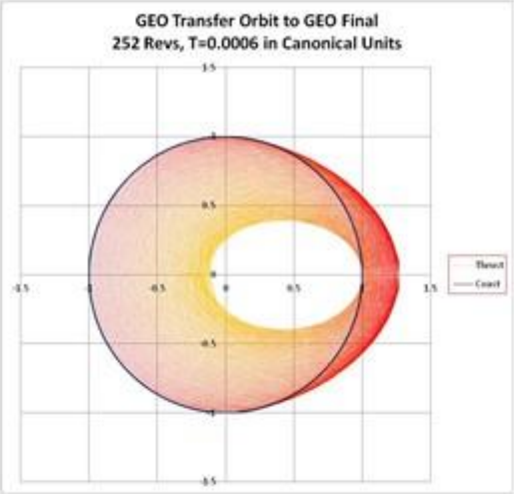




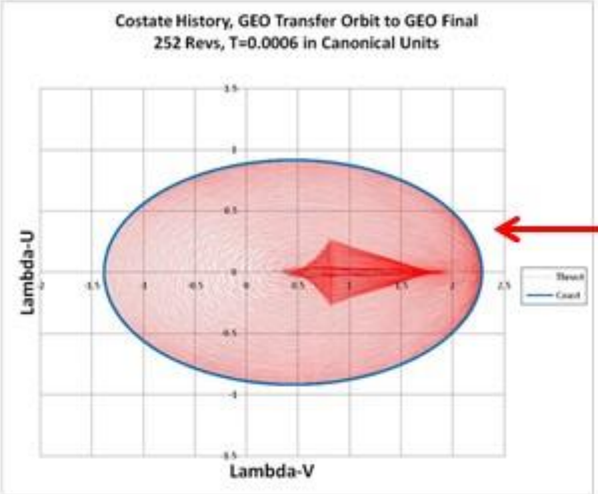


GTO to GEO Transfer

Physical Trajectory

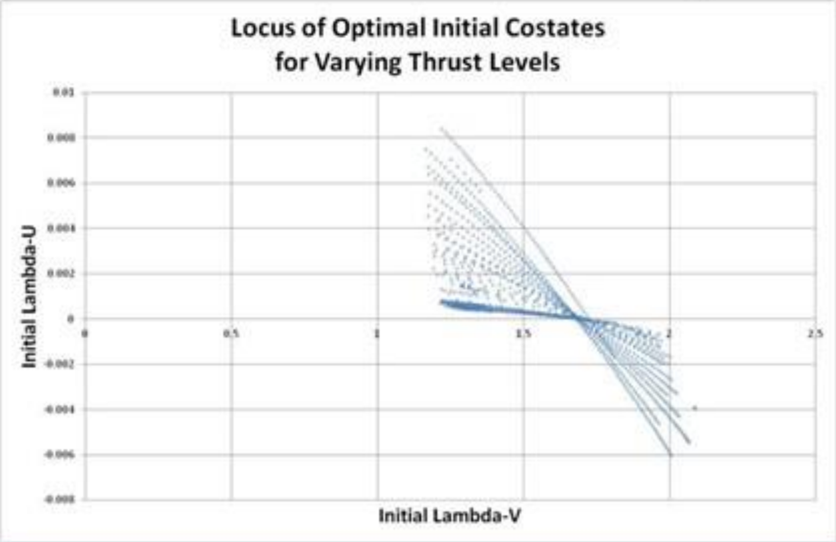


Control Vector History



Path crossings imply possible local minima

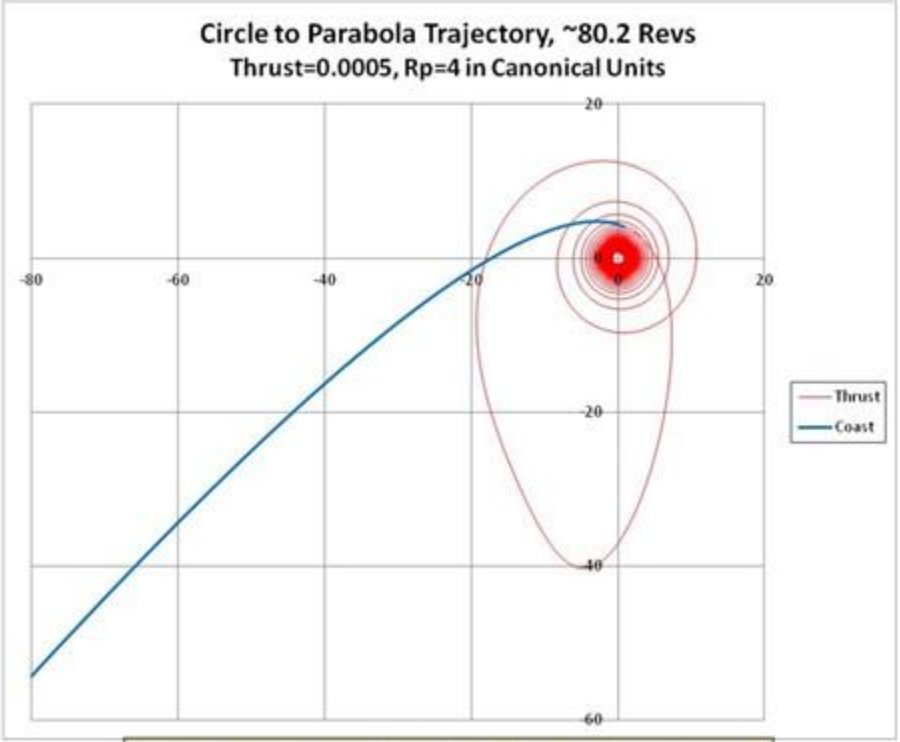
Initializing the Optimal Control Law



Escape Velocity Example

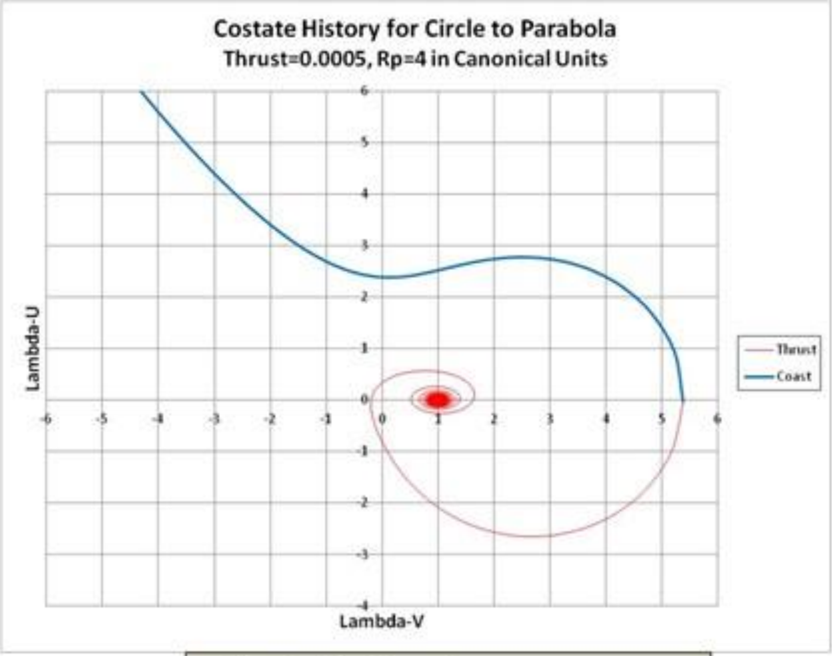
Circular to Parabolic – Produced by Indirect Optimization

Physical Trajectory



Similar to Impulsive Bi-Elliptic Transfers

Control Vector History

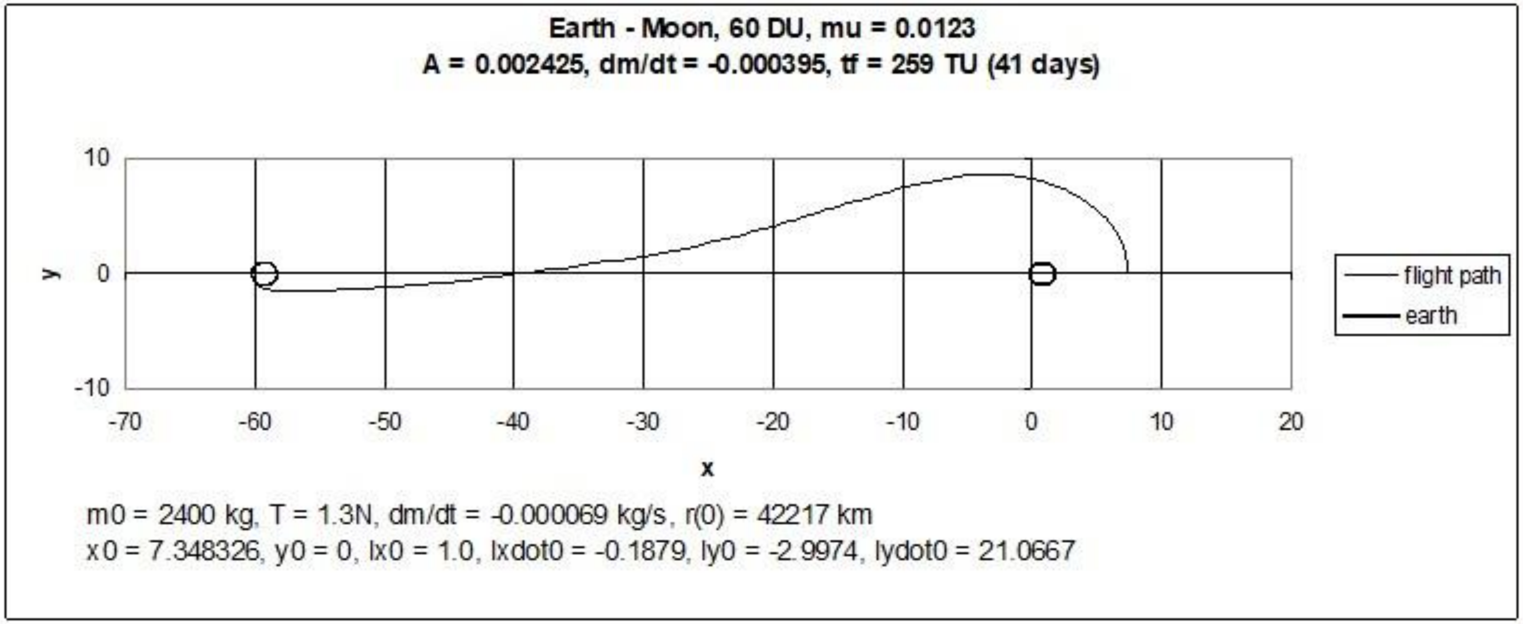


- No Path Crossings
- No Local Minima over Thrust
- No Discontinuities over Thrust

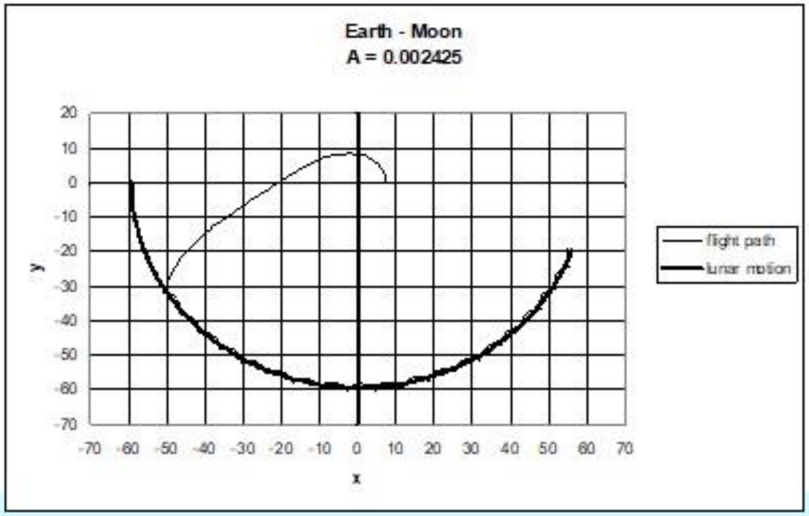
Red arcs are thrusting, blue arcs are coasting

Earth to Moon 3-Body Example
Transfer from GEO to Lunar Orbit

Rotating Frame



Inertial Frame



Summary

- Classical indirect methods can optimize complex orbital transfers without the use of direct collocation methods, because all of the trajectory information is contained in the initial values of the costates

- Starting from an ellipse with multiple revolutions will result in discontinuities in parameter searches, but these can be overcome by modeling the initial costate patterns and estimating accumulated velocity change to guide the parameter search

- Starting from a smaller circular orbit to any final condition appears to result in a unique, minimum-time solution trajectory

- **Checking for path crossings of the costate histories can predict the existence of local minima versus unique solutions**

Questions?

Numerical Example – Europa Science Orbit

Europa, circle-circle

init inc = 0 deg, final inc = 70 deg and 120 deg

μ = 3201 km^3/s^2

r1 = (1560.8+100) km (Europa's radius + initial orbit altitude)

r2 = 35,000 km (final orbital radius about Europa)

1 DU = 1660.8km

1 TU = 1196.28s

1 MU = 25,000 kg

μ = 1 DU^3/TU^2

r1 = 1 DU, r2 = 21.0742 DU

acc = 0.0033 and 0.01 DU/TU^2, mdot = 0

3D Thrust angles:

alpha measured from x in x,y; beta "up" from x,y plane to position vector, range: +/- 180 deg